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A generalized Milne-Thomson theorem

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Abstract

Using analytic continuation theory, a new simple proof of a standard generalized circle theorem is given. Additionally, new cases involving complex coefficients in the boundary condition and allowing for an arbitrary singularity of a given complex potential at the interface are considered.

Keywords: Heterogeneous media; Circular theorem; Analytic functions

1. Introduction

It is well known that physically different phenomena are frequently described by the same mathematical laws when formulated as models in terms of partial differential equations and boundary conditions. In this work one such standard model, that of the theory of heterogeneous media, is considered. This classical model can be described as follows.

It is required to define a two-dimensional planar stationary field $\mathbf{v}(x, y) = (v_x, v_y) = \mathbf{v}_p(x, y)$, $(x, y) \in S_p$, $p = 1, \ldots, m$, which is potential and solenoidal in each isotropic phase S_p of an m-phase medium:

$$\operatorname{div} \mathbf{v}_p(x, y) = 0, \qquad \operatorname{curl} \mathbf{v}_p(x, y) = 0, (x, y) \in S_p,\ p = 1, \ldots, m. \tag{1.1}$$

It is assumed that the continuous limit boundary values of the vectors $\mathbf{v}$ and $\hat{\rho}\mathbf{v}$ satisfy the conditions

$$[\mathbf{v}_p(x, y)]_n = [\mathbf{v}_q(x, y)]_n, \qquad [\hat{\rho}_p \mathbf{v}_p(x, y)]_\tau = [\hat{\rho}_q \mathbf{v}_q(x, y)]_\tau, (x, y) \in \mathcal{L}_{pq}, \tag{1.2}$$

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